Applicability of Redundancy Calibration to Dense Phased Arrays

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Abstract. We aim to have an efficient and computationally cheap calibration method for dense phased arrays or any array which has enough redundant baselines. The most recently developed calibration method, multisource calibration requires a sky model. This requires computational capacity of the array’s processor, especially when an extended structure such as the galactic plane is present. This is especially the case because of the very short baselines. Redundancy calibration is independent of a sky model. This is the main reason why we study this method, in particular because dense phased arrays have many redundant baselines. In this paper, we present initial results of redundancy calibration on dense phased arrays. The results are sufficiently promising to keep us motivated to continue to pursue this method in more details.

Keywords: dense phased array, calibration, redundancy, multisource, mutual coupling.

1. Introduction

A hierarchy of calibration levels is required for the next generation of radio telescopes as explained in [1]. Calibration of phased arrays or station calibration is an important step in this hierarchy. Its goal is to track the variations of the complex electronic gains of the receivers over time and frequency. A robust calibration together with beamforming should guarantee a stable beam pattern of the station for the central correlator. This is crucial for high fidelity imaging after the central correlator.

In phased arrays, correlation between all elements can be calculated. These correlations include many short baselines on which a partially resolved sky is captured. The most recent calibration method for the phased arrays is multisource calibration introduced by Wijnholds and Van der Veen [2]. The multisource calibration method needs the presence of some relatively unresolved point sources such as Cas A and a model of the extended structures. However, at a given time, detection of known sources is not guaranteed. Moreover, modeling the extended structures is hard and computationally expensive.

Dense phased arrays operating above ~ 100 MHz are often implemented as compound elements or tiles, such as HBAs (High Band Antenna) at the LOFAR stations or EMBRACE (Electronic Multi-Beam Radio Astronomy ConcEpt). Having a regular antenna arrangement gives the possibility of having redundant baselines i.e. baselines with the same physical length and orientation (see Figure 2). This motivated us to try redundancy calibration instead. This method is independent of the sources in the sky. Its basic idea is that we should capture the same visibilities on the redundant baselines [5, 6]. It uses the data of all redundant baselines to obtain a convergent calibration solution.

In the following, we will build up a data model on the basis of which the two calibration methods can briefly be introduced.

Afterwards the initial results of redundancy calibration on HBA data will be presented and discussed. The redundancy calibration method is not new but its application to dense phased arrays is novel. Therefore, there are some further steps left to take.

2. Methods

We will first discuss the data model and then describe in detail the multisource and redundancy calibration methods.

2.1. data model

Let’s assume that we have a phased array of p elements. Then we can express the array signal vector, \( x(t) = [x_1(t), x_2(t), ..., x_p(t)]^T \) as:

\[
x(t) = \Gamma \Phi \left( \sum_{k=1}^{q} a_k s_k(t) \right) + n(t) = \Gamma \Phi s(t) + n(t)
\]

where \( s(t) \) is a \( q \times 1 \) vector containing \( q \) mutually independent i.i.d.¹ Gaussian signals impinging on the array. They are also assumed to be narrow band, so we can define the matrix \( \Phi \) as:

\[
\Phi = \left[ e^{j \theta_1}, e^{j \theta_2}, ..., e^{j \theta_q} \right]^T
\]

Correspondingly, the matrix \( \Gamma \) is defined as:

\[
\Gamma = \text{diag}(\gamma)
\]

The amplitudes and phases of the independent complex gains which have to be calibrated are \( \gamma = [\gamma_1, \gamma_2, ..., \gamma_p]^T \) and \( \theta = [\theta_1, \theta_2, ..., \theta_q]^T \).

1. independent (over time) and identically distributed.
Then the model for the visibility matrix describing the correlations between all elements can be written:

\[
\mathbf{R} = \mathbf{\Gamma} \mathbf{\Phi} \mathbf{\Sigma}_r \mathbf{\Phi}^H \mathbf{\Gamma}^H + \mathbf{\Sigma}_n
\]

(2)

where \(\mathbf{\Sigma}_r\) and \(\mathbf{A}\) are assumed to be known. We can calculate them, if we specify the time of observation, the telescope geometry and known source parameters.

In dense phased arrays such as the LOFAR HBA and EMBRACE, the tiles are tightly packed. This may cause mutual coupling between the tiles. It is an important and frequency dependent effect on amplitudes and phases of the observed visibilities. It can be shown that the mutual coupling effect can be included as a matrix multiplication in our data model. Referring to [3] and [4], a first order approximation for the mutual coupling matrix \(\mathbf{M}\) can be assumed using Eq. 3, to define the off diagonal elements and \(M_{ii} = 1\) as the diagonal elements.

\[
M_{ij} = -m_a A_i r_{ij}(1 - m_b) \cos(\phi_{ij})) \exp(2\pi j r_{ij}/\lambda)
\]

(3)

where \(m_a\) and \(m_b\) are constant values, \(r_{ij}\) is the distance between two elements, \(\lambda\) represents the frequency, \(\phi_{ij}\) shows the orientation of the baseline between the two elements. Having a model of mutual coupling, we can simply disentangle it from the observed data.

### 2.2. Multisource calibration method

The multisource calibration problem can be formulated as a least squares minimization problem:

\[
[\hat{\mathbf{g}}, \hat{\sigma}_r] = \arg \min_{\mathbf{g, \sigma}_r} \| \mathbf{R} \mathbf{\Phi} \mathbf{\Sigma}_r \mathbf{\Phi}^H \mathbf{\Gamma}^H + \mathbf{\Sigma}_n - \hat{\mathbf{R}} \|_F^2
\]

(4)

This estimates the noise and complex gain of each receiver element using the measured visibility, \(\hat{\mathbf{R}}\) and the modeled visibility, \(\mathbf{\Gamma} \mathbf{\Phi} \mathbf{\Sigma}_r \mathbf{\Phi}^H \mathbf{\Gamma}^H + \mathbf{\Sigma}_n\). In [2], the mathematical solutions to this problem are comprehensively discussed. Here we just emphasize that the initial and important assumptions that let the method lead to a straightforward solution are:

- The receivers’ noise are uncorrelated and accordingly \(\mathbf{\Sigma}_n\) is diagonal.
- Complex gains are either direction-independent or direction-dependent and known. This direction dependency can be absorbed in the known sky \(\mathbf{\Sigma}_r\).
- The impinging signals are narrow-band. This means we can represent time delays by phase shifts.

Figure 1 shows a sky map scanned by HBA tiles. One can see the galactic plane due to the many short baselines and the sun as the dominant radio source. To apply the multisource calibration on HBA data, we need to have the corresponding data model; model of the extended structure and an unstable source like the sun.

### 2.3. Redundancy calibration method

The redundancy calibration method was introduced by Noordam and De Bruyn in 1982 [5]. It has successfully been applied to WSRT (Westerbork Synthesis Radio Telescope) observations since then. Its basic idea is very simple but clever; theoretically, the visibilities on redundant baselines are the same. We call those true visibilities. What we actually measure are the true visibilities multiplied by antenna and baseline dependent gains perturbed by additive errors and noise [6]:

\[
R_{ij} = R_{ij}^{true} G_i G_j + c_{ij} + e_{ij}
\]

(5)

where \(G_{ij}\) is baseline dependant complex gain, \(c_{ij}\) is additive error due to e.g. correlator offset and \(e_{ij}\) is the thermal noise plus possible interference. If we assume that errors in \(G_{ij}\) as well as \(c_{ij}\) are negligible, we obtain:

\[
R_{ij}^{obs} = R_{ij}^{true} G_i G_j + e_{ij}
\]

(6)

By taking the natural logarithm from both sides of the Eq. 6, we obtain individual linear equations for the amplitude and phases of complex values:

\[
r_{ij}^{obs} = r_{ij}^{true} + g_i + g_j + a_{ij}
\]

(7)

\[
\psi_{ij}^{obs} = \psi_{ij}^{true} + \phi_i - \phi_j + b_{ij}
\]

(8)

Note that even for Gaussian noise \(e_{ij}\), the error terms \(a_{ij}\) and \(b_{ij}\) will have complicated distributions that depend on the SNR. For the rest of this paper, we assume a high SNR. So we can ignore the error terms. The resulting set of linear equations written for all the redundant baselines can be solved using a single step least squares method. This estimator will converge, if we add some constraints based on the actual situation of the array. We set those constraints to the best of our knowledge. Since we have to specify the absolute flux level, we set:

\[
\Sigma g_i = 0
\]

(9)
We also have to constrain the absolute element phase. We can enforce this constraint by specifying that the average phase for all elements is zero:

$$\Sigma \phi_i = 0$$  \hspace{1cm} (10)

There might also be an arbitrary linear phase slope over the array. This phase slope corresponds to a position shift of the field. This arises because redundancy cannot determine an absolute position. This can either be absorbed in the true visibilities or in the element phases. It can be shown, this is the null space of the matrix we have built up by Eq. 8. Since we have a two dimensional array unlike WSRT, we constrain x and y in the same manner:

$$\Sigma_{i=1}^p \phi_i x_i = 0$$ \hspace{1cm} (11)

$$\Sigma_{j=1}^p \phi_j y_j = 0$$ \hspace{1cm} (12)

This method is independent of a sky model. Instead it requires: a) enough SNR to do a meaningful comparison between the redundant visibilities, b) enough redundancy in the array to get all the elements involved in the system of equations.

3. Discussion

Independency of redundancy calibration from a sky model is a strong reason why we investigate this method. In dense phased arrays like HBA and EMBRACE, the tiles are set in a regular arrangement. This provides a significant number of redundant baselines. We studied the performance of redundancy calibration using different data sets. The presented results are on data captured on 26th May 2009 at 13:12:40 UTC using RCU mode 5 (frequency 110-190 MHz). The visibility in each subband was integrated over one second.

Figure 2 shows 36 different types of redundant baselines that are available in a 24-tile HBA station. We present the result of redundancy calibration on the type indexed 10. Referring to Eq. 7 and Eq. 8, the parameters to be estimated are the true visibilities and amplitudes and phases of the complex receiver gains. In Figure 3 the left panels show the amplitudes and phases of the observed visibilities. One can clearly see that the visibilities (in both amplitude and phase) are redundant. Regarding the mutual coupling, we can certainly say that its effect is very small in the case of HBA data. Based on the model given in Eq. 3, its effect on redundant baselines of the same type by different elements is different. Therefore if it were a large effect, we wouldn’t see what we see now in the left panels. The right panels show how the method estimates amplitude and phase of the true visibility using the data in the left panels. Deviation from the observed values is a few percent. Phase wrapping is seen in the lower right panel. This is a standard problem that we still have to solve.

Figure 4 shows how the method estimates amplitudes of complex receiver gain. The elements number 9, 10, 15 and 16 are chosen for this. Under normal circumstances, we expect this value to vary smoothly over frequency. One can see that its variance is a few percent.

Figure 5 shows how the method estimates phases of complex receiver gain for the same elements. This value is also supposed to vary smoothly over frequency. But we can not justify the way these phases vary. We may have to reconstrain the phases in a different way. This in addition to the phase wrapping problem requires further work.

4. Conclusion and further work

Herein we presented the initial results of the redundancy calibration method on real data of a dense phased array. The results are sufficiently promising to make us believe this method is not only computationally cheap but also statistically efficient. Therefore it is potentially the method of choice for calibrating dense phased arrays such as the HBA at LOFAR stations, EMBRACE and hopefully dense phased arrays in SKA. These results demonstrate that one should seriously consider the advantages of redundancy when considering the final SKA layout and technology.

We just started exploring this method for the next generation of radio telescopes. There still is a lot to be examined and tested. Some of the major steps forward are:

1. The redundancy calibration method should be applied on simulated data. This helps us to improve or change the constraints we set at the beginning.

2. The redundancy calibration method should be evaluated mathematically as an estimator; Monte-Carlo simulation and CRLB (Cramer-Rao Lower Bound) evaluation are required. In this way, we can compare it with the existing calibration methods.

3. We should study how its precision depends on the strength of RFI sources and SNR of the data.

4. We should also quantify the errors resulting from the different assumptions that we make.

References


Fig. 2. 36 different redundant baselines indicated on the layout of a 24-tile HBA station.

Fig. 3. Upper left: the amplitude of the observed visibilities. Upper right: the estimated amplitude for the true visibility. Lower left: the phase of the observed visibilities. Lower right: the estimated phase for the true visibility.
Fig. 4. Amplitude of complex receivers’ gains. Out of 24 elements, the elements no. 9, 10, 15 and 16 are chosen.

Fig. 5. Phase of complex receivers’ gains. Out of 24 elements, the elements no. 9, 10, 15 and 16 are chosen.